COMPOUND INTEREST AND ANNUITY TABLES

COMPOUND INTEREST AND ANNUITY TABLES 8 Percent							
NO. OF YEARS HENCE	COM-POUND	PRESENT VALUE OF $1.00	AMORTIZ ATION	VALUE OF AN ANNUITY - ONE PER YEAR		PRESENT VALUE OF AN ANNUITY	
				Present	Future	Increasing	Decreasing
1	1.08	0.926	1.080	0.926	1.000	0.926	0.926
2	1.17	0.857	0.561	1.783	2.080	2.641	2.709
3	1.26	0.794	0.388	2.577	3.246	5.022	5.286
4	1.36	0.735	0.302	3.312	4.506	7.962	8.598
5	1.47	0.681	0.250	3.993	5.867	11.365	12.591
6	1.59	0.630	0.216	4.623	7.336	15.146	17.214
7	1.71	0.583	0.192	5.206	8.923	19.231	22.420
8	1.85	0.540	0.174	5.747	10.637	23.553	28.167
9	2.00	0.500	0.160	6.247	12.488	28.055	34.414
10	2.16	0.463	0.149	6.710	14.487	32.687	41.124
11	2.33	0.429	0.140	7.139	16.645	37.405	48.263
12	2.52	0.397	0.133	7.536	18.977	42.170	55.799
13	2.72	0.368	0.127	7.904	21.495	46.950	63.703
14	2.94	0.340	0.121	8.244	24.215	51.717	71.947
15	3.17	0.315	0.117	8.559	27.152	56.445	80.507
20	4.66	0.215	0.102	9.818	45.762	78.908	127.273
50	46.90	0.021	0.082	12.233	573.770	151.826	472.081
100	2,199.76	0.000	0.080	12.494	27,484.516	168.105	1,093.821

Interest and annuity tables provide a reference to enable the user to properly account for the effects of interest and time in making an economic analysis. The basic principles of the time value of money, and the use of interest factors in making comparisons between values that occur at different points in time are presented.

Interest and annuity problems have four elements in common: (a) an amount, (b) an interest rate, (c) a term, and (d) a payment. If any three of these elements are known, then the fourth can be derived from the tables.

Procedures for discounting future benefits and costs or otherwise converting benefits and costs to a common time basis are also presented.

BASIC DEFINITIONS

Value ______________ In economics, value represents any quantity expressing the worth of something. In resource development projects, value is used to express benefits arising from effects of project measures, or it could be the cost for providing such measures.

Number of Years Hence — This is the number of periods (years, months, or days) in which calculations are considered. There may be many conditions which influence this determination: (1) a benefit may last a year or indefinitely (perpetuity), (2) the measures may have a short or long useful life, (3) the period of evaluation may be set by policy, (4) an individual may want to recover his costs in a certain time period, or (5) costs or returns may occur over varying time periods or at varying rates *for* the same period.

Annuity — Annuity is a series of equal payments made at equal intervals of time. The most common type of annuity is our paychecks, at least those that meet the equal payment requirement. Annuity may be a benefit (to those receiving equal sums of money) or a cost (to those making the payment).

Interest — Interest is economic rent of money. When money is borrowed, the amount borrowed must be repaid along with a use charge called interest. Or, said another way, interest is money paid for the use of money.

The appropriate rate of interest will depend upon the situation or the reason for the analysis. Demand, time, and risk (includes inflation) determine the rate of interest charged or paid in commercial lending establishments. If personal money is used in lieu of borrowed or lent funds, an opportunity cost[1] should be taken into consideration.

Interest rate is expressed as a percent of the principal amount and is understood to be an interest rate per year. There are two kinds of interest--simple and compound

Simple Interest — Simple interest is rent on the principle amount only. If $100 is loaned or borrowed and a year later $108 is repaid, the $8 is interest and, since it was for one year, the interest rate is 8 percent. The amount of interest is computed by the following formula:

i= (p)(r)(n), where i = interest, p = principle ($100), r = periodic interest rate (8%), and n = number of periods (1 year).

i = ($100)(.08)(1) = $8.00

Compound Interest — Compound interest is interest that is earned for one period and immediately added to the principle, yielding a larger principle on which interest is computed for the following period. This means the accrued unpaid interest is actually converted to additional principle.
Problem: What will $500 grow to in 5 years at 8 percent interest?
Solution: ($500)(1.46933[2]) = $734.67

The following represents the compound interest factor
Formula:

$(1 + i)^n$, where n is the number of periods, i is the periodic rate of interest, and 1 represents one dollar since the formula results in a factor that is multiplied by the principle dollar amount.

NOTE: Compound interest factors are not shown by column heading in the tables, but the same answer can be obtained by dividing the appropriate "present value *of* 1" factor since the present value *of* 1 factor is the reciprocal *of* the compound interest factor. Using the preceding problem:

[1] 0pportunity cost is the return forgone from the most likely alternative use of money.
[2] Compound interest, 5 years hence, and 8 percent interest (not shown in tables).

What will $500 grow to in 5 years at 8 percent interest? The solution is as follows:

Solution: $500 / .68058[1] = $734.67

Present Value of 1

Sometimes called present worth of 1 is what $1.00 due in the future is worth today or at present. The present value of a specified single sum of money due at some named future date is that sum of money which, if put at compound interest for the same time period would have a compound amount equal to the specified amount. Hence, this is the reason for the factor being occasionally called the "discount factor." Delayed cost or benefits can be reduced to present worth at year 0 with this factor.

Problem: At 8 percent interest find the present value of $1,000 to be received in 5 years.

Solution: ($1,000)(.68058[2]) = $680.58

In other words, $1,000 to be received in 5 years is worth $680.58 today.

The present value of 1 factor is represented by the following formula.

$$\frac{1}{(1 + 1)^n}$$

NOTE: The present value of 1 factor is the reciprocal of the “compound interest” factor.

Amortization

Amortization is sometimes called partial payment or capital recovery factor. This factor will convert capital or initial cost to annual cost. It will determine what annual payment including interest must be made to payoff the initial cost over a given number of years.

Problem: At 8 percent interest, find the annual equivalent investment cost over a period of 10 years of a facility with an initial cost of $1,000.

Solution: Annual cost = initial cost multiplied by the amortization factor for 10 years at 8 percent interest.

Annual cost = ($1,000)(.14903[3])

Annual cost = $149.03

The formula for the amortization factor is expressed as:

$$\text{Amortization} = \frac{i\,(1 + i)^n}{(1 + i)^n - 1} \quad \text{or.} \quad \frac{i}{1 - \frac{1}{(1 + i)^n}}$$

NOTE: The amortization factor is the reciprocal of the "present value of an annuity of 1 per year" factor which means that the same answer can be obtained by dividing by the present value of an annuity of 1 per year factor. For example, using the above problem the solution is as follows:

$1,000 / 6.7100[4] = $149.03

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[1] Present value of 1, 5 years hence, 8 percent interest.
[2] Present Value of 1, 5 years hence, 8 percent interest.
[3] Amortization, 10 years hence, 8 percent interest.
[4] Present value of an annuity of 1 per year, 10 years hence, 8 percent interest.

Present Value of an Annuity of 1 Per Year

Present value of an annuity of 1 per year also referred to as constant annuity, present worth of an annuity, or capitalization factors.

This factor represents the present value or worth of a series of equal deposits over a period of time. It tells us what an annual deposit of $1.00 is worth today. If a fixed sum is to be deposited or earned annually for "n" years, this factor will determine the present worth of those deposits or earnings.

Problem: If $600 will be placed in your savings account each year for 10 years, what is the present worth of the total amount or if you want to give $600 per year to someone for 10 years what sum would be required at present? The interest rate is 8 percent.

Solution: $600 x 6. 71008[2] = $4,026

This is the present value of receiving $600 per year *for* 10 years or it is the amount that would need to be deposited today to make annual withdrawals of $600 for 10 years.

The present value of an annuity of 1 per year factor is expressed as follows:

$$\frac{(1 + i)^{n} - 1}{1\,(1 + i)^{n}}$$

NOTES:

a. The factor is the reciprocal of the "amortization" factor. Therefore, the same answer can be obtained by dividing by the amortization factor:

Solution: $600 / .14903[1] = $4,026

b. "Amount of an annuity of 1 per year" multiplied by "present value of 1" = "present value of an annuity of 1 per year."

Amount of an Annuity of 1 Per Year

Amount of an Annuity of 1 per year is also called "accumulation of an annuity." As stated earlier an annuity is a sequence of equal payments made at uniform intervals with each payment earning compound interest during it's respective earning term. The amount of an annuity of 1 per year factor shows how much an annuity, invested each year, will grow over a period of years. It can also show how much it is worth to provide protection against losing the opportunity of investing an annuity each year. For example, preventing erosion that causes a loss of net income of $25 per year for 50 years has an accumulated value of $14,344 at the end of 50 years at 8 percent interest (25 x 573.77016[1]).

The factor for an amount of an annuity of 1 per year is expressed as follows:

$$\frac{(1 + 1)^{n} - 1}{1}$$

Another example is a person establishing a retirement reserve. If $500 is placed in a savings account, each year earning 8 percent, what is the amount of the reserve at the end of 20 years?

Solution: ($500)(45. 76196[2]) = $22,881

[1] Amount of an annuity of 1 per year, SO years hence, and 8 percent interest.
[2] Amount of an annuity of 1 per year, 20 years, 8 percent interest

Sinking Fund

This factor is used to determine what size annual deposit will be required to accumulate to a certain amount (given) in a certain number (given) of years at compound interest.

Problem: If $25,000 is needed to meet a term note due in 10 years, what amount will need to be deposited each year at 8 percent compound interest to reach the goal?

Solution: ($25,000)(.069029[1]) = $1,726

The sinking fund factor is expressed as:

$$\text{Sinking Fund} = \frac{1}{(1 + i)^{n} - I}$$

NOTE: The sinking fund factor is not shown by column heading in the tables but the same answer can be obtained by dividing by the appropriate "amount of an annuity of 1 per year" factor since the amount of an annuity of 1 per year factor is the reciprocal of the sinking fund factor.

Solution: $25,000 / 14.48656[2] = $1,726

Present Value of an Increasing Annuity

This factor shows how much something is presently worth that will provide increasing sums of money over a period of years.

It should be noted that to meet the definition of an annuity (equal payments at equal intervals) the increases must be uniform. For example, seeding a field to grass may eventually be worth $18 per acre in increased net income, but it will take 6 years before this value is realized. The annuity is increasing $3 per year (18 / 6) so the definition of an annuity is satisfied. The total annuity we will receive is not level or the same each year since

We will receive $3 the first year, $6 the second year, and $9 the third year until at the end of the sixth year the $18 per year will be realized. At 8 percent interest the present value or present worth of something that will provide this amount of flow or build up of funds is $45.44 ($3 x 15.14615[3]).

The factor for "present value of an increasing annuity" is expressed in the following formula:

$$\frac{(1 + i)^{n+1} - (1 + I) - n(i)}{(1 + i)^{n} (i)^{2}}$$

Present Value of a Decreasing Annuity

This factor tells us how much something is presently worth that will provide an annuity that gets less each year.

The decrease must be uniform to meet the definition of an annuity. For example, a sediment pool will return $1,000 recreation benefits the first year but at the end of 40 years, because of sediment accumulation, it will have no value.

The flow of funds is decreasing uniformly at $25 per year ($1,000 / 40 years). Therefore, the basic definition of annuity is met. Benefits would be $1,000 the first year, $975 the second year, $950 the third year, etc., until at the end of 40 years there would be no returns.

[1] Sinking fund factor 10 years hence, 8 percent interest (not shown in tables).
[2] Amount of an annuity of 1 per year, 8 percent interest, 10 years hence.
[3] Present value of an increasing annuity, 6 years hence, 8 percent interest.

The present value of this sediment pool at 8 percent interest is $8,774 ($25 x 350.94233[1]). The $8,774 invested at 8 percent interest would return the decreasing annuities described.

The factor for "present value of a decreasing annuity" is displayed in the following formula:

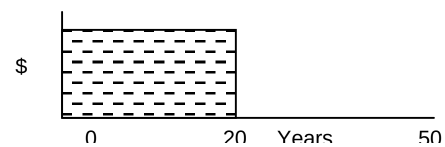
$$\frac{n\,(i) - 1 + \dfrac{1}{(1 + I)^n}}{(1 + i)^n\,(i)^2}$$

Clues to Problem Solving by the use of Diagrams & Sketches

The solution to complicated problems is made easier by the use of a sketch or diagram. There are six basic diagrams that can be combined or used to illustrate all problems involving decreases, increases, delays, or discount by the use of the compound interest and annuity tables. By using the graphic illustrations and lettering the basic parts, all parts of the more complicated problems can be broken down into a series of simple familiar steps, each one easy to understand and solve. You will also save time and make fewer mistakes.

Clue 1

Situation: A return of $600 will be received each year for 20 years. What is the average annual value for a 50-year evaluation period at 8 percent interest?



Step 1 - Determine present worth of benefits occurring evenly over each of the 20 years. Multiply $600 by the factor for the "present value of annuity of 1 per year" corresponding to the length of return (20 years).

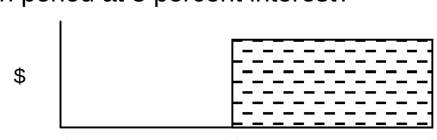
($600)(9.81815)[2] = $5,891

Step 2 - Determine the annual equivalent value of the income stream. Multiply the $5,891 by the amortization" factor corresponding to the length of the evaluation period (50 years).
($5,891)(.08174)[3] = $482 \

Conclusion: The average annual value is $482.

Clue 2

Situation: A return of $600 will be received for 20 years. However, it will be 30 years before the return will begin. What is the average annual value for a 50-year evaluation period at 8 percent interest?



Step 1 - Determine present worth of benefits occurring evenly over each of the 20 years (50-30). Multiply $600 by the factor for "present value of an annuity of 1 per year" corresponding to the length of return (20 years).

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[1] Present value of decreasing annuity, 40 years, and 8 percent interest.
[2] Present value of an annuity of 1 per year, 20 years, and 8 percent interest.
[3] Amortization, 50 years, 8 percent interest.

(600)(9.81815[1]) = $5,891

Step 2 - Discount the benefits that are delayed 30 years. Multiply the $5,891 by the discount factor "present value of 1" corresponding to the length of delay (30 years).

($5.891)(.09938[2]) = $585

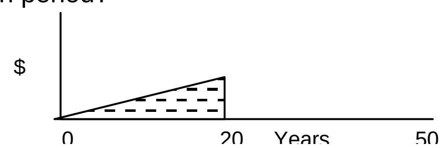
Step 3 - Determine the annual equivalent value of the capital sum. Multiply the $585 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($585)(.08174[1]) = $48

<u>Conclusion:</u> The average annual value is $48.

Clue 3 ______________________

Situation: A return will begin to build from $0 in year 0 to $600 in 20 years then cease. What is the average annual benefit at 8 percent interest for a 50-year evaluation period?



Step 1 - To meet the definition of an annuity where values are uniformly increasing, divide the ending value reached by the number of years to reach the value.

$$\frac{\$600}{20} = \$30 \text{ per year}$$

Step 2 - Determine present worth of the income stream. Multiply the $30 per year by the "present value of an increasing annuity" corresponding to the number of years the return is received (20 years).

($30) (78.90794[3]) = $2,367

Step 3 -Determine annual equivalent value of the income stream. Multiply the $2,367 by the “amortization" factor corresponding to the length of the evaluation period (5O years).

($2.367)(.08174[4]) = $193

<u>Conclusion:</u> The average annual benefit is $193.

Clue 4 ______________________

Situation: A return will begin to build from $0 in year 30 to $600 in year 50. What is the average annual benefit at 8 percent interest for a 50-year evaluation period?

[1] Present. value *of* an annuity of 1 per year, 20 years, and 8 percent interest.
[2] Present value of 1, 30 years, 8 percent interest.
[3] Present value of an increasing annuity of 1 per year, 20 years, -8 percent interest
[4] Amortization. 50 years, 8 percent interest.

Step 1 - To meet the definition of an annuity where values are uniformly increasing, divide the ending value reached by the number of years to reach the value.

$$\frac{\$600}{20} = \$30 \text{ per year}$$

Step 2 - Determine present worth of the income stream beginning in year 30. Multiply the $30 per year by the "present value of an increasing annuity" corresponding to the number *of* years the return is received (20 years).

($30)(78.90794[1]) = $2,367

Step 3 -Discount the income stream that is delayed 30 years. Multiply the $2,367 by the discount factor (present value of 1) corresponding to the length of delay (30 years).

($2,367)(.09938[2]) = $235

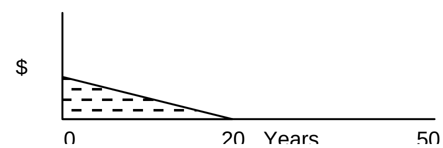
Step 4 - Determine the annual equivalent value of the income stream. Multiply the $235 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($235)(.08174[1]) = $19

Conclusion: The average annual benefit is $19

Clue 5

Situation: A present return of $600 will uniformly decrease to $0 at year 20. What is the average annual value of 8 percent interest rate for a 50-year evaluation period?



Step 1 -To meet the definition of an annuity where values are uniformly decreasing, divide the beginning value by the number of years to reach $0.

$$\frac{\$600}{20} = \$30 \text{ per year}$$

Step 2 - Determine the present worth of the income stream. Multiply the $30 per year by the "present value of a decreasing annuity" corresponding to the number of years the return is to be received (20 years).

($30)(127.27316[3]) = $3,818

Step 3 - Determine the annual equivalent value of the income stream. Multiply the $3,818 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

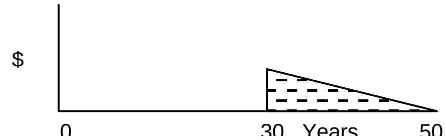
($3.818)(.08174[4]) = $312

Conclusion: The average annual benefit is $312.

[1] Present value of an increasing annuity, 20 years, 8 percent interest.
[2] Present value of 1, 30 years, 8 percent interest
[3] Present value of a decreasing annuity, 20 years, 8 percent interest.
[4] Amortization, 50 years, 8 percent interest.

Clue 6 ______________________

Situation: After year 30, return of $600 uniformly decreases to $0 at year 50. What is the average annual value at 8 percent interest rate?



Step 1 - To meet the definition of an annuity where values are uniformly decreasing, divide the beginning value by the number of years to reach $0.

$$\frac{\$600}{20} = \$30 \text{ per year}$$

Step 2 - Determine present worth of the income stream beginning in year 30. Multiply the $30 per year by the "present value of a decreasing annuity" corresponding to the number of years the return is received (20 years).

($30)(127.27316[1]) = $3,818

Step 3 - Discount the income stream that's delayed 30 years. Multiply the $3,818 by the discount factor (present value of 1) corresponding to the length of delay (30 years).

($3,818)(.09938[3]) = $379

Step 4 -Determine the annual equivalent value of the income stream. Multiply the $379 by the "amortization" factor corresponding to the length of the evaluation period (50 years).

($379)(.08174[2]) = $31

Conclusion: The average annual value is $31

[1] Present value of 1, 30 years, 8 percent interest

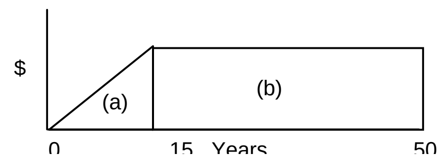
Example Problems

The following problems illustrate the use of the compound interest and annuity tables. The purpose of the problems is to illustrate the principles used in applying each of the annuity factors described in the previous section

Problem 1

A net return will increase in uniform amounts to $3,000 for a 15-year period. Thereafter, through the remaining 35 years of the evaluation period, the benefits will remain constant at 8 percent interest. Determine the annual equivalent value over the 50-year evaluation period.

First diagram the problem.



Solution:

a. $3,000 / 15 = $200 per year increase
($200)(56.44514[1]) = $11,289

b. ($3,000)(.31524[2])(11.65457[3]) = $11,022

Annual Equivalent Value: $11,289 + $11,022(/ $22,311
($22,311)(.0817[4]) = $1,824

Shortcut -Table A (page 14) provides straight-line lag discount factors that can be used directly for selected percents of interest.

The illustration of the shortcut method in solving the above problem is as follows:

($3,000)(0.608[5]) = $1,824

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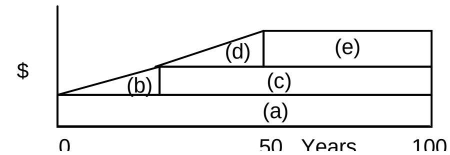
1 Present value of an increasing annuity, 15 years hence, 8 percent interest
2 Present value of 1I, 15 years hence, 8 percent interest
3 Present value of an annuity of 1 per year, 35 years hence, 8 percent interest.
4 Amortization, 50 years, 8 percent interest.
5 Table I, page 14,15 year lag, 50-year evaluation period, and 8 percent interest.

Problem 2

Land use will become more intensive in a flood prevention-drainage project as the operators accept and utilize the protection made possible by the project. Presently net benefits per acre are $50. They are expected to uniformly increase to $95 per acre in 20 years. Thence, to $149 by the end of 50 years and then remain uniform for the remainder of the project life. Determine the average value per acre if the interest rate is 8 percent and the evaluation period is 100 years.

Diagram the problem:



Solution:

a. ($50)(12.494321[1]) = $625

b. $95 -$50 = $45

45 / 20[2] = $2.25 per year

($2. 25)(78.90794[3]) = $178

c. ($45)(12.47351[4])(.21455[5]) = $120

d. $149 -$95 = $54

54 / 03[6] = $1.80 per year

($1.80)(114.71358[7])(.21455[5]) = $44

e. ($54)(12.23348[8])(.02132[9]) = $14

The present value of receiving the above benefits is $981 (625 + 178 + 120 + 44 + 14).

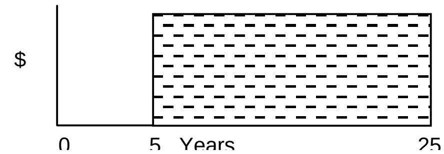
The average annual value per acre is $78.52 (981 x .08004[10])

[1] Present value of an annuity of 1 per year, 100 years, and 8 percent interest.
[2] Necessary to meet the definition of an annuity.
[3] Present value of an increasing annuity, 20 years, 8 percent interest.
[4] Present value of an annuity of 1 per year, 80 years, and 8 percent interest.
[5] Present value of 1, 20 years, 8 percent interest.
[6] Necessary to meet the definition of an annuity, (50 -20) = 30 years.
[7] Present value of an increasing annuity, 30 years, 8 percent interest.
[8] Present value of an annuity of 1 per year, 50 years, and 8 percent interest.
[9] Present value of 1, 50 years, 8 percent interest.
[10] Amortization, 100 years, 8 percent interest.

Problem 3

It will take 5 years for seeding on rangeland to become established after which it will yield a value of $500 on a continuing basis. What is the average annual benefit for the 25-year evaluation period at 8 percent interest?

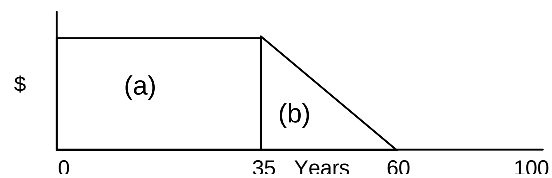
Diagram the problem:



Solution: ($500)(9.81815[1])(.68058[2])(.09368[3]) = $313

Problem 4

The recreational use of a reservoir is expected to be 3,000 visits the first year of use and will remain constant for 35 years. They are then expected to decline to 0 at the 60th year of the 100-year evaluation period. Using an 8 percent interest rate, determine the average annual recreational benefits if visits are valued at $1.25 each.



Solution: a. (3,000}($l.25) = $3,750

(3,750)(11.65457[4]) = $43,705

b. $3,750 / 25[5] = $150 per year

($150)(179.06530[6])(.06763[7]) = $1,817

$43,705 + $1,817 = $45,522

(45,522}(.08004[8]) = $3,644

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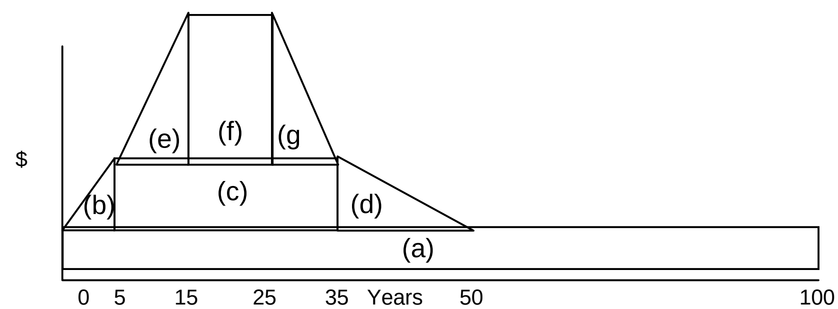
[1] Present value of an annuity of 1, 20 years, 8 percent interest.
[2] Present value of 1, 5 years, 8 percent interest.
[3] Amortization, 25 years, 8 percent interest.
[4] Present value of an annuity of 1 per year, 35 years, and 8 percent interest.
[5] Necessary to meet the definition of an annuity. (60 years - 35 years = 25 years.)
[6] Present value of a decreasing annuity, 25 years hence, 8 percent interest.
[7] Present value of 1, 35 years hence, 8 percent interest.
[8] Amortization, 100 years, 8 percent interest.

Problem 5

This problem combines all of the previous principles into one example. Assume this problem involved benefits that would accrue in the following manner: begin with $300; grow to $800 in 5 years; grow to $1500 by year 15; remain constant until year 25, then decrease to $800 by year 35; decrease to $300 by year 50 and remain constant to year 100.

To solve this complicated problem, first break the problem into segments that can be easily computed.



a. Is a constant annuity[1] for the duration of the evaluation period.

b. Is a buildup period or increasing annuity for 5 years with no lag or delay.

c. Is a constant annuity[1] for the duration of the evaluation period.

d. Is a decreasing annuity for 15 years delayed 35 years.

e. Is an increasing annuity for 10 years delayed 5 years.

f. Is a constant annuity[1] for 10 years delayed 15 years.

g. Is a decreasing annuity for 10 years delayed 25 years.

Dollar values listed in the problem and the factors from the tables are multiplied together. The values for (a) through (g) are then added together and amortized over the evaluation period to determine the average annual values. Remember where increasing and decreasing annuities are involved, you must divide the dollar value by the number of years, to get an annual rate of increase or decrease.

[1] Constant annuity is referred to in the tables as "present value of an annuity of 1 per year."

Solution: The average annual value at 8 percent interest is as follows:

a. (\$300)(12.49432[1])	=	\$3,748
b. \$800 - \$300 = \$500		
\$500 / 5[2] = \$100		
(\$100)(11.36514[3])	=	\$1,137
c. (\$500)(11.25778[4])(68058[5])	=	\$3,831
d. \$500 / 15[2] = 33.33		
(33.33)(80.50652[6])(.06763[7])	=	\$181
e. \$1.500 - \$800 = \$700		
\$700 / 10[2] = \$70		
(\$70)(32.68691[8])(.68058[5])	=	\$1,557
f. (\$700)(6.71008[9])(.31524[10])	=	\$1,481
g. \$700 / 10 = \$70		
(\$70)(41.1239[11])(.14602[12])	=	\$420

(\$3,748 + \$1,137 + \$3,831 + \$181 + \$1,557 + \$1,481 + \$420 = \$12,355

(\$12,355)(.08004[13]) = \$989 Average Annual Value

[1] Present value of an annuity of 1 per year, 100 years hence, 8 percent interest.
[2] Necessary to meet the definition of an annuity
[3] Present value of an increasing annuity, 5 years hence, 8 percent interest.
[4] Present value of an annuity of 1 per year, 30 years hence, 8 percent interest.
[5] Present value of 1, 5 years, 8 percent interest.
[6] Present value of a decreasing annuity, 15 years hence, 8 percent interest.
[7] Present value of 1, 35 years hence, 8 percent interest.
[8] Present value of an increasing annuity, 10 years hence, 8 percent interest.
[9] Present value of an annuity of 1 per year, 10 years hence, 8 percent interest.
[10] Present value of 1, 15 years, 8 percent interest.
[11] Present value of a decreasing annuity, 10 years hence, 8 percent interest.
[12] Present value of 1, 25 years, 8 percent interest.
[13] Amortization, 100 years, 8 percent interest.